

Testing Dark Matter with the Anomalous Magnetic Moment in Quantum Electrodynamics

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We consider a model of dark quantum electrodynamics which is coupled with a visible photon through a kinetic mixing term. After checking of consistency properties, we compute the $g_w - 2$, where g_w is the gyromagnetic factor for a dark fermion. The $g_w - 2$ and $g - 2$ of quantum electrodynamics are related by the kinetic mixing factor. We analyse the $g_w - 2$ in terms of the ratio $\kappa = m_\gamma/m_\chi$ where m_γ and m_χ are the masses of the dark photon and the dark fermion and we discuss how light and heavy fermions become very different for $m_\gamma \leq 10^{-5}$ eV. This analysis is also carried out using different available data.

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I. INTRODUCTION

The search of dark matter is one of the most important challenges of physics because their existence may explain many puzzles of conventional physics, such as the rotation curves of the galaxies, new phenomena of emission of light of the center of galaxies or to provide new insights to old problems such as matter-antimatter asymmetry, primordial magnetic fields and so on. Dark matter, of course could also open new fields in particle physics, astrophysics and cosmology [1–3].

The essential issue is to find phenomena that can be measured with high precision, as for example in atomic physics and QED [4] and then compare with similar phenomena that may occur in the dark sector of the universe. In this context a good example is the scattering of an electron in an external field (for example the Coulomb potential). This problem is interesting in itself because historically it led to the Rutherford model of the atom in the early days of the quantum theory and together with this fact, one can ask, how are the dark atoms? This question is much more than purely academic because the notion of “macroscopic” dark matter body is only a way of thinking to parameterise the differences in the galaxies rotation curves by using the Kepler laws, thus the dark matter concept is simply a way to adjust an observational data by adding new parameters that conventionally, we call dark matter.

Although in the current state of knowledge there is no single way to understand this fundamental problem, there are several equivalent approaches that help us to understand at least formally these issues. One of these ways is based in the description of effective Hamiltonian (Lagrangian) where one chooses by hand the terms visible and dark in the Hamiltonian and using criteria that depend on the problem under consideration. A good example of this procedure was used in [5–8] with the Hamiltonian ¹

$$H_{eff} = -\mathbf{d} \cdot \mathbf{E} = -d \frac{\mathbf{S}}{S} \cdot \mathbf{E}, \quad (1)$$

where the electric dipole \mathbf{d} moment is parallel to the spin of the dark fermion. An “almost” obvious fact is that the dimension of \mathbf{d} is m_χ^{-1} where m_χ is the mass of dark fermion. What is assumed here is that the fermion (with spin) is dark but the electric field is visible which is also the criterion that we will use.

Now let us assume that the electric field is static so that it can be written in terms of the scalar potential as $\mathbf{E} = -\nabla A_0$. This allows us to rewrite (1)

$$H_{eff} = -d_\chi \frac{\mathbf{S}}{S} \cdot (-\nabla A_0). \quad (2)$$

Here we have let $\mathbf{d} \rightarrow \mathbf{d}_\chi$ in order to emphasise that \mathbf{d}_χ is the electric dipole moment of the dark particle. The exact value of \mathbf{d} depends on the unknown mass scale m_χ of the dark fermion.

In the momentum space, we can replace $\nabla \rightarrow i\mathbf{q}$ in the effective Hamiltonian, where \mathbf{q} is the momentum of the (visible) photon ($\hbar = 1$). Similarly, the scalar potential A_0 represents the Coulomb potential and in the momentum space it is given by $A_0 = \frac{4\pi Ze}{|\mathbf{q}|^2}$. Therefore, in momentum space the effective Hamiltonian operator becomes [5]

$$\hat{H}_{eff} = \mathbf{S} \cdot i\mathbf{q} \left(\frac{d_\chi}{S} \frac{4\pi Ze}{|\mathbf{q}|^2} \right). \quad (3)$$

In the interaction picture, the effective Hamiltonian leads to the differential cross section

$$d\sigma = \left| \langle a | \hat{H}_{eff} | b \rangle \right|^2 d\Omega = |A_{ba}|^2 d\Omega,$$

where $A_{ba} = \langle a | \hat{H}_{eff} | b \rangle$ is the transition amplitude from the state $|b\rangle$ to the state $|a\rangle$.

If the initial and the final states, $|a\rangle$ and $|b\rangle$, are spin-averaged, the differential cross section yields

$$d\sigma = \frac{\text{Tr}(S_i S_j)}{S^2} \frac{q_i q_j}{|\mathbf{q}|^4} d_\chi^2 (4\pi)^2 Z^2 e^2 d\Omega. \quad (4)$$

Furthermore, using the identity for particles with spin S

$$\text{Tr}(S_i S_j) = (2S + 1) \delta_{ij} \frac{S(S + 1)}{3},$$

¹ Here we follow [5].

equation (4) leads to

$$\frac{d\sigma}{d\Omega} = \frac{(2S+1)(S+1)}{3S} \frac{(4\pi d_\chi Z e)^2}{|\mathbf{q}|^2}. \quad (5)$$

This is essentially the result obtained in [5] and although this result is physically very nice, the relationship between visible and dark matter is still unclear and we would like to develop this idea further using a different point of view.

II. DARK MATTER AND KINETIC MIXING

In order to understand this problem systematically, let us build our model of *dark QED*. First, we consider a dark fermion χ with mass m_χ coupled to a hidden photon B_μ . The coupling constant (for minimal coupling) of dark fermions to the hidden photon is taken to be unity $e_h = 1$ for simplicity so that the covariant derivative in the dark sector can be written as $D_\mu[B] = \partial_\mu + iB_\mu$. We note that since we are interested in electric dipole interactions, we have to couple the dark fermion non-minimally and we choose the Lagrangian density for the dark fermion to be given by

$$\mathcal{L}_\chi = \bar{\chi} \left(i \not{D}[B] + \frac{g_\chi}{\Lambda} \sigma_{\mu\nu} F^{\mu\nu}(B) - m_\chi \right) \chi. \quad (6)$$

The Pauli coupling may arise from radiative corrections or can be included in the tree level Lagrangian density in an effective theory. Here Λ is a mass scale which, in the case of (visible) QED, is proportional to the electron mass m and therefore, by analogy, can be chosen to be proportional to the mass of the dark fermion (m_χ). The constant g_χ can be thought of as the gyromagnetic factor in analogy with standard QED. We point out that $\sigma_{\mu\nu} F^{\mu\nu}$ in the Pauli term can be written in terms of the electric and magnetic fields as

$$\frac{1}{2} \sigma_{\mu\nu} F^{\mu\nu} = -\boldsymbol{\sigma} \cdot \text{diag} \{ (\mathbf{B}_h + i\mathbf{E}_h), \boldsymbol{\sigma} \cdot (\mathbf{B}_h - i\mathbf{E}_h) \}, \quad (7)$$

and therefore the Dirac equation interacting non-minimally coupled to a gauge field B_μ contains both an electric dipole term $\boldsymbol{\sigma} \cdot \mathbf{E}$ and a magnetic dipole term $\boldsymbol{\sigma} \cdot \mathbf{B}$.

Next, we choose the dynamics for the hidden photon given by the Lagrangian density

$$\mathcal{L}_B = -\frac{1}{4} F_{\mu\nu}(B) F^{\mu\nu}(B) + \frac{m_B^2}{2} B_\mu B^\mu, \quad (8)$$

where m_B is the mass of the dark photon which can arise, for example, through the Higgs mechanism or the Stückelberg formalism in the hidden sector. Similarly, in the visible sector the dynamics of the photon can be given by the standard Maxwell term. However, we will assume the kinetic mixing model of [9] which allows for a mixing between the gauge bosons in the visible and the hidden sectors through the Lagrangian density

$$\mathcal{L}_{KM} = -\frac{1}{4} F_{\mu\nu}(A) F^{\mu\nu}(A) + \frac{\xi}{2} F_{\mu\nu}(A) F^{\mu\nu}(B). \quad (9)$$

Here ξ is a dimensionless mixing parameter assumed to be small and we can also add a term which represents the interaction of the photon with the visible charged current [10]. However, we neglect this term for simplicity since it is not relevant to our analysis. The complete Lagrangian density for our system, therefore, is given by

$$\mathcal{L} = \mathcal{L}_\chi + \mathcal{L}_B + \mathcal{L}_{KM}. \quad (10)$$

To extract physics from this Lagrangian we can eliminate the kinetic mixing term [11] by redefining $A'_\mu = \sqrt{1 - \xi^2} A_\mu$, $B'_\mu = B_\mu - \xi A_\mu$ and $\xi' = \frac{\xi}{\sqrt{1 - \xi^2}}$, which leads to (from now on $F^2 = F_{\mu\nu} F^{\mu\nu}$)

$$\mathcal{L} = -\frac{1}{4} F^2(A') - \frac{1}{4} F^2(B') + \frac{m_B^2}{2} (B' + \xi' A')^2 + \bar{\chi} \left(i \not{\partial} + \not{B}' + \xi' \not{A}' - m_\chi \right) \chi + \mathcal{L}_P, \quad (11)$$

where, with this redefinition, the Pauli term \mathcal{L}_P has the form

$$\mathcal{L}_P = \frac{g_\chi}{\Lambda} \bar{\chi} \left(\sigma_{\mu\nu} F^{\mu\nu}(B') + \xi' \sigma_{\mu\nu} F^{\mu\nu}(A') \right) \chi. \quad (12)$$

For $\Lambda = 8m_\chi$, we see that g_χ in the Pauli interaction (12) can be thought of as the gyromagnetic factor of the dark matter particle whose magnetic moment is given by

$$\boldsymbol{\mu} \equiv -\frac{g_\chi}{2m_\chi} \mathbf{S},$$

with \mathbf{S} , the spin, in complete analogy with standard QED. However, the second term

$$\frac{g_\chi \xi'}{8m_\chi} \bar{\chi} \sigma_{\mu\nu} F^{\mu\nu}(A') \chi,$$

represents an interaction of dark matter to visible photons. Indeed, let us evaluate the previous expression in the static limit of a constant (visible) magnetic field \mathbf{B} which leads to

$$\frac{g_\chi \xi'}{8m_\chi} \bar{\chi} \sigma_{\mu\nu} F^{\mu\nu}(A') \chi \xrightarrow{\text{static } \mathbf{A}} \bar{\chi} \left[\left(-\frac{g_\chi \xi'}{2m_\chi} \mathbf{S} \right) \cdot \mathbf{B} \right] \chi.$$

The coefficient $g'_\chi \equiv g_\chi \xi'$ plays the role of a gyromagnetic factor in the interaction of dark matter with visible photons. This is interesting because although g_χ is an unknown parameter as it originates in processes in the dark sector of the theory, the second term involves the interaction with the visible photon opening up doors for its measurement.

The one loop quantum vertex corrections originating from both the terms involve the two diagrams shown in Figure 1. Fig. 1(a) denotes the vertex diagram describing the gyromagnetic factor in the dark sector, while Fig. 1(b) shows the gyromagnetic factor for the interaction with the photon in the visible sector. The detailed calculations for these are described in the following section.

A. Details of the Calculation

We have already seen that in our model we can identify two gyromagnetic factors. One of them (g_χ) is a genuine correction to the magnetic moment of the dark fermion obtained from diagram Fig. 1(a), while the other comes from the interaction with the visible photon, as is shown in Fig. 1(b). The integrands for the two diagrams are the same and, therefore, the calculations can be done in the conventional manner [12]. Even though this calculation is well known, we will describe this in some detail in order to clarify differences with the standard case.

The effect of loop contributions to the vertex is equivalent to modify $\gamma_\mu \rightarrow \Gamma_\mu$ where the new vertex function can be written as

$$\Gamma^\mu = \gamma^\mu F_1 \left(\frac{p^2}{m_\chi^2} \right) + \frac{\sigma^{\mu\nu}}{2m_\chi} p_\nu F_2 \left(\frac{p^2}{m_\chi^2} \right). \quad (13)$$

Here, F_1 and F_2 are the form factors which depend on the Lorentz invariant combination (p^2/m_χ^2) of the transferred momentum p_μ and the mass scale of the particle m_χ . Then the diagram in Fig. 1(a) gives rise to the amplitude

$$i\mathcal{M}^\mu = -i\bar{u}(q_2) \left[F_1 \left(\frac{p^2}{m_\chi^2} \right) \gamma^\mu + \frac{\sigma^{\mu\nu}}{2m_\chi} p_\nu F_2 \left(\frac{p^2}{m_\chi^2} \right) \right] u(q_1), \quad (14)$$

where q_1 is the four momentum of the incoming particle and q_2 , the outgoing one. The four moment of the external photon is $p = q_2 - q_1$. Diagram in Fig. 1(b) gives rise to a similar contribution except that the coupling of the dark fermion with visible photon has an extra factor ξ' , as can be seen from the Lagrangian density (11). The two terms appearing in (14) have well known physical interpretations. While F_1 contributes to charge renormalization, F_2 provides a genuine contribution to the magnetic moment. In fact, the gyromagnetic factor g can be obtained from here through

$$g = 2(1 + F_2(0)). \quad (15)$$

Form factors can be calculated from the diagrams as follows. The amplitude for the diagram in Fig. 1(a) is

$$i\mathcal{M}^\mu = -\bar{u}(q_2) \int \frac{d^4k}{(2\pi)^4} \frac{\eta_{\alpha\beta} \gamma^\alpha (\not{p} + \not{k} + m_\chi) \gamma^\mu (\not{k} + m_\chi) \gamma^\beta}{[(k - q_1)^2 - m_\gamma^2 + i\varepsilon][(p + k)^2 - m_\chi^2 + i\varepsilon][k^2 - m_\chi^2 + i\varepsilon]} u(q_1). \quad (16)$$

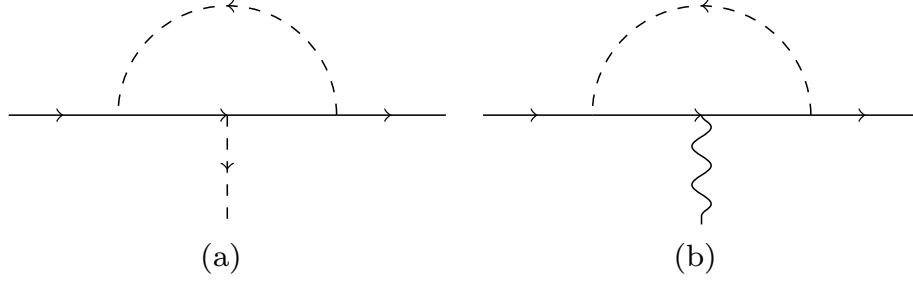


FIG. 1: The two possible vertex corrections involved in the gyromagnetic factor of dark matter (dashed lines represent hidden photons propagators). The diagram a) is the genuine contribution to the magnetic moment coming from the coupling of the dark fermion to the hidden photon. Diagram b), on the other hand, corresponds to the contribution coming from the coupling of the dark fermion to the visible photon. Both diagrams differ by a factor of ξ' .

After some algebra (long but straightforward) one can identify contributions proportional to the operator $\sigma^{\mu\nu}p_\nu$, which we are interested in (see (14)), to correspond to

$$F_2^{(a)}(p^2) = -i 8 m_\chi^2 \int_0^1 dx dy dz \delta(x+y+z-1) \int \frac{d^4 k}{(2\pi)^4} \frac{z(1-z)}{(k^2 - \Delta + i\varepsilon)^3} + \dots \quad (17)$$

where $\Delta = -xyp^2 + (1-z)^2 m_\chi^2 + m_B^2 z$.

The integral over k can be calculated in the usual way [12] and leads to

$$F_2^{(a)}(p^2) = \frac{1}{8\pi^2} \int_0^1 dx dy dz \delta(x+y+z-1) \frac{z(1-z)}{(1-z)^2 + \kappa^2 z - xy \frac{p^2}{m_\chi^2}}. \quad (18)$$

with $\kappa^2 = m_B^2/m_\chi^2$. This is the standard result of QED if the photon is massive which is the case for our hidden photon.

In the limit $p_\mu \rightarrow 0$, the form factor turn out to be

$$F_2^{(a)}(0) = \frac{1}{8\pi^2} \int_0^1 dx dy dz \delta(x+y+z-1) \frac{z(1-z)}{(1-z)^2 + \kappa^2 z}. \quad (19)$$

In the limit $\kappa \rightarrow 0$ (massless photons), the value of the integral is $1/2$ and, in this case, $2F_2(0) = \alpha/2\pi$ (if we reinsert the factors of charge e^2 which we have set to unity in the hidden sector) which coincides with the Schwinger gyromagnetic factor $g-2$ [4]. Furthermore, for diagram 1(b), which gives rise to magnetic moment of the dark fermion through coupling to the visible photon, the calculation is completely parallel and we can write the results for the contributions from the two diagrams (to magnetic moment) as

$$F_2^{(a)}(0) = \frac{1}{8\pi^2} f(\kappa), \quad F_2^{(b)}(0) = \frac{\xi'}{8\pi^2} f(\kappa). \quad (20)$$

The function $f(\kappa)$ can be obtained from (19) and a direct calculation gives

$$f(\kappa) = -\frac{\kappa(2-4\kappa^2+\kappa^4)}{\sqrt{4-\kappa^2}} \tan^{-1} \left(\frac{\sqrt{4-\kappa^2}}{\kappa} \right) + \frac{1}{2} [1-2\kappa^2+2\kappa^2(\kappa^2-2) \ln \kappa]. \quad (21)$$

It may appear that this function is defined only for $0 < \kappa < 2$. However, the result is, in fact, valid for all values of $\kappa > 0$. This can be seen directly by noting that imaginary parts appearing in \tan^{-1} cancel with those coming from the square root in the denominator. Figure 2(a) illustrates this behavior.

The gyromagnetic factors coming from the diagrams a) and b) (g_χ and g'_χ) turn out to be

$$g_\chi - 2 = 2 F_2^{(a)} = f(\kappa), \quad (22)$$

$$g'_\chi - 2 = 2 F_2^{(b)} = \xi' f(\kappa), \quad (23)$$

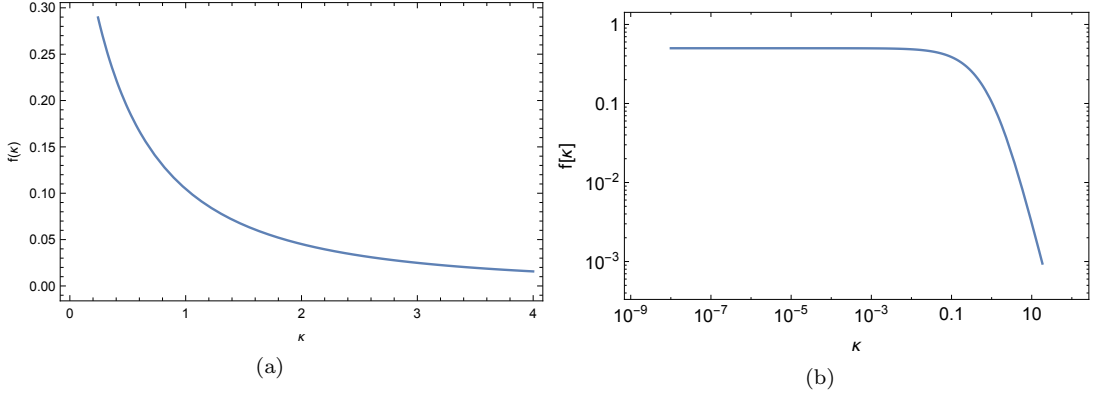


FIG. 2: Panel (a) shows the behaviour of $f(\kappa)$ vs κ for a wide range of values of κ . Panel (b) shows the behaviour of $f(k)$ in a log-log plot, showing a zone of fast decreasing for $\kappa \sim 2$. This feature can be appreciated in the following figures.

implying the anticipated identity

$$g'_\chi - 2 = \xi'(g_\chi - 2), \quad (24)$$

as has been mentioned earlier.

A final comment is in order here. A different diagram, which can also contribute to the gyromagnetic factor of the dark fermion when it couples to the visible photon, comes from diagram Fig. 1(b) when the internal hidden photon propagator is replaced by the visible photon. This diagram is allowed from the interactions in (11), but it is suppressed by a factor of ξ'^2 which is very small [14] and we are discarding terms of this order in this calculation.

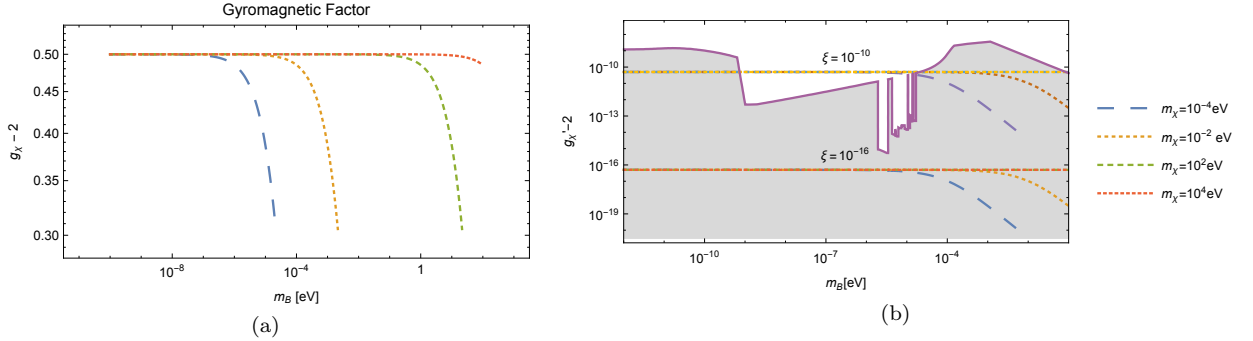


FIG. 3: Diagram (a) shows values of $g_\chi - 2$ as a function of the mass of the hidden photon (m_B), for different values of the mass of the dark fermion (m_χ), in a log-log plot. Right diagram is the log-log plot of the gyromagnetic factor $g'_\chi - 2$ (that is when dark fermion couples to visible photons) as a function of m_B , for two different values of the coupling constant ξ' (in the approximation $\xi' \sim \xi$).

Possible values of $g_\chi - 2$ factor are shown in Figure 3(a). As it is expected from the shape of $f(\kappa)$ (in the left diagram of Figure 2), as the mass of the dark fermion becomes larger, the value of the gyromagnetic factor approaches the value of the gyromagnetic factor of the electron only if the electric charge of the hidden fermion is equal to the electric charge of the electron. This behavior is expected because for heavy dark fermions the mass of the hidden photon can be neglected (remember $\kappa = \frac{m_B}{m_\chi}$) which coincides with the visible sector (zero mass photons). In this case, one obtains the same gyromagnetic factor provided the hidden electric charge is equal to the electron charge. It is also interesting to note that for low mass dark fermions, the value of $g_\chi - 2$ decreases very fast for specific values of m_B . This effect is generic for a fixed value of $\frac{m_B}{m_\chi}$ as it can be seen from Fig. 2(b).

Finally, we can study the behaviour of the $g'_\chi - 2$ factor from Fig. 3(b) (we are taking $\xi' \sim \xi$). In order to do that, we use the allowed bounds for the parameters ξ and m_B which can be found in [10, 13, 14]. Since $g'_\chi - 2$ depends linearly on ξ' ($\xi' \approx \xi$), the shape we observe in Fig. 3(b) is very similar to that in Fig. 3(a). In Fig. 3(b) we have filled allowed region of the parameter space (ξ, m_B) including the exclusion dark matter region given by Helioscope measurements [15]). Note that values $\xi \leq 10^{-15}$ are not ruled out yet by any observation and then we have to take

values bounded for such quantity in order to render the gyromagnetic factor (for the coupling with visible sector) a universal feature of dark particles.

III. FINAL COMMENTS AND CONCLUSIONS

In this paper the calculation of the magnetic moment of dark fermion was made by taking, as starting point, a model of hidden quantum electrodynamics coupled to visible photons through a kinetic mixing term [9]. The kinetic mixing term provides a link where dark and visible are related and an explicit relation between g_χ and g'_χ was found (eq. (24)). From the analysis of (24) along with comparison with the data of measurements, we conclude that g_χ changes drastically if fermions are light, otherwise the same quantum electrodynamics behavior is obtained if the electric charge of dark matter is the same as the electron charge. It is quite possible that in other applications of quantum electrodynamics, such as atomic physics test and pair production, one may find interesting results.

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